Name:________________________________

MA 3053 Section 01 — *Practice Exam 1* — *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

The definitions you must absolutely know for the exam:

(a) A function, $f : X \to Y$

(b) A function being injective

(c) Composition of two functions

(d) The pre-image/image of set under a function

(e) The arbitrary union/intersection of sets

(f) Equivalence relation on a set

(g) The specific equivalence relation, congruence mod n, and the congruence classes of these

1. Let $f : X \to X$. Suppose f has the property, $f \circ f = \mathrm{id}|_X$. That is $(f \circ f)(x) = x$ for all $x \in X$. Show f is an injection.

2. Let $f : X \to Y$. Given functions $g, h : W \to X$ such that whenever $f \circ g = f \circ h$, then $g = h$; show that f is injective.

3. Let $f : X \to Y$ and $V_\alpha \subseteq Y$ for every $\alpha \in A$ Show

$$f^{-1}\left(\bigcup_{\alpha \in A} V_\alpha\right) = \bigcup_{\alpha \in A} f^{-1}(V_\alpha)$$

4. Let $f : X \to Y$ and $V_\alpha \subseteq X$ for every $\alpha \in A$ Show

$$f\left(\bigcup_{\alpha \in A} V_\alpha\right) = \bigcup_{\alpha \in A} f(V_\alpha)$$

5. Let $\sim$ be a relation on $X = \mathbb{Z} \times \mathbb{N}^+$ by $(a, b) \sim (c, d)$ if and only if $ad = bc$. Show $\sim$ is an equivalence relation on X.

6. Let $\sim$ be a relation on $\mathbb{R}$ by $x \sim y$ if and only if $|x| = |y|$. Show $\sim$ is an equivalence relation on $\mathbb{R}$.

7. What are the multiplication and addition tables for the congruence classes in $\mathbb{Z}/7\mathbb{Z}$.

8. What are the multiplication and addition tables for the congruence classes in $\mathbb{Z}/4\mathbb{Z}$.

9. Let $f : X \to Y$ and $V_\alpha \subseteq X$ for every $\alpha \in A$. Show that

$$f\left(\bigcap_{\alpha \in A} V_\alpha\right) \subseteq \bigcap_{\alpha \in A} f\left(V_\alpha\right)$$